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Phase Properties of New Even and Odd Nonlinear Coherent States

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Using the Pegg–Barnett formalism of phase operator, we obtain phase probability distributions of new even and odd nonlinear coherent states. It is shown that the distributions for the states are rather different, and unlike the case of ordinary even and odd coherent states the Pegg–Barnett distribution clearly reflects the different character of quantum interference in the case of the new even and odd coherent states.

KEY WORDS: new even and odd nonlinear coherent states; Pegg–Barnett formula; phase probability distribution.

1. INTRODUCTION

The idea of nonlinear coherent states (NLCS) has existed in the quantum optics literature for many years, although in the earlier history other names for these states were frequently used (for a useful summary, see Dodonov, 2000). Generally speaking the NLCS may be defined as right eigenstates of deformed annihilation operator A. The deformation may take various forms, but a typical case is $A = af(N)$, where a is a standard harmonic oscillator annihilation operator and the deforming function $f(N)$ is an operator-valued real function of the number operator $N = a^+a$ (de Matos Filho and Vogel, 1996; Man'ko *et al.*, 1997). These NLCS exhibit nonclassical features like squeezing and self-splitting. A class of NLCS can be realized physically as the stationary states of the center-of-mass motion of a trapped ion (de Matos Filho and Vogel, 1996). On the basis of the work (Man'ko *et al.*, 1997), the concept of even and odd NLCS were constructed

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by Mancini (1997). A kind of orthogonal even NLCS was introduced by Das (2000). Recently, a new kind of NLCS was constructed by Roy and Roy (2000) (referred as Roy-type NLCS hereafter). On the basis of this work, we defined a new type of even and odd NLCS (Wang *et al*., 2003), i.e., the even and odd Roy-type NLCS, and investigated the quantum statistical properties of the states, including quadrature squeezing, amplitude-squared squeezing, and antibunching effect. It may be noted that the phase probability distribution is an essential tool in the study of various phase characteristics. In the present paper, using the Pegg–Barnett formalism (Barnett and Pegg, 1989; Pegg and Barnett, 1988, 1989) of phase operator, we discuss the phase probability distributions of the even and odd Roy-type NLCS. It is shown that the distributions for the even and odd Roy-type NLCS are rather different, and unlike the case of ordinary even and odd coherent states the Pegg–Barnett distribution clearly reflects the different character of quantum interference in the case of the even and odd Roy-type NLCS.

2. DEFINITION OF THE EVEN AND ODD ROY-TYPE NLCS

For convenience of reference and completeness, in this section we begin with some related results for the NLCS (Man'ko *et al*., 1997) and the Roy-type NLCS (Roy and Roy, 2000).

The NLCS $|\alpha, f\rangle$ are defined as right eigenstates of the generalized annihilation operator $A = af(N)$ (de Matos Filho and Vogel, 1996; Man'ko *et al*., 1997):

$$A|\alpha, f\rangle = \alpha|\alpha, f\rangle, \tag{1}$$

where α is a complex number. In the number state basis, the NLCS $|\alpha, f\rangle$ is given by

$$|\alpha, f\rangle = C\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}\,f(n)!}|n\rangle, \quad C = \left\{\sum_{n=0}^{\infty}\frac{|\alpha|^{2n}}{n![f(n)!]^2}\right\}^{-1/2}, \tag{2}$$

where $f(n)! = f(n)f(n-1)\ldots f(1)f(0)$ and $f(0) = 1$.

The canonical conjugate of the generalized annihilation and creation operators A and A^+ are given by Roy *et al*. (2000)

$$B = a\frac{1}{f(N)}, \qquad B^+ = \frac{1}{f(N)}a^+. \tag{3}$$

In the number state basis, the Roy-type NLCS (Roy and Roy, 2000) are defined as the right eigenstates of the new generalized annihilation operator B,

$$|\beta, f\rangle = N_f\sum_{n=0}^{\infty}\frac{\beta^n f(N)!}{\sqrt{n!}}|n\rangle, \quad N_f = \left\{\sum_{n=0}^{\infty}\frac{|\beta|^{2n}[f(n)!]^2}{n!}\right\}^{-1/2}, \tag{4}$$

where β is an arbitrary complex number.

In the our previous paper (Wang *et al.*, 2003), we followed the definition of the even and odd NLCS (Mancini, 1997) (i.e., the eigenstates of the operator A^2) and defined a new kind of even (+) and odd (−) NLCS (called even/odd Roy-type NLCS) in a straightforward manner as

$$|\beta,\ f\rangle_{\pm} = N_{\pm}(|\beta,\ f\rangle \pm |-\beta,\ f\rangle), \quad N_{\pm} = \left\{2 \pm 2N_f^2 \sum_{n=0}^{\infty} \frac{(-|\beta|^2)^n [f(n)!]^2}{n!}\right\}^{-1/2}. \tag{5}$$

By using Eqs. (4) and (5), the even and odd Roy-type NLCS can be expanded in the number basis as

$$|\beta,\ f\rangle_{\pm} = N_{\pm} N_f \sum_{n=0}^{\infty} \frac{[r^n \pm (-r)^n]\, e^{in\varphi} f(n)!}{\sqrt{n!}} |n\rangle, \tag{6}$$

where $\beta = r\exp(i\varphi)$.

3. PHASE PROBABILITY DISTRIBUTIONS OF THE EVEN AND ODD ROY-TYPE NLCS

In this section we shall examine the phase probability distributions of the even and odd Roy-type NLCS given by Eq. (6). However, before we proceed any further, it is necessary to specify the nonlinearity function $f(n)$. From Eq. (6), it is clear that for every choice of $f(n)$ we shall have the different even and odd Roy-type NLCS. In the present case we choose the following nonlinearity function which has been used in the description of the motion of a trapped ion (de Matos Filho and Vogel, 1996):

$$f(n) = L_n^1(\eta^2)\left[(n+1)L_n^0(\eta^2)\right]^{-1}, \tag{7}$$

where η is known as the Lamb–Dicke parameter and $L_n^m(x)$ are generalized Laguerre polynomials (Abramowitz and Stegun, 1972). Clearly, $f(n) = 1$ when $\eta = 0$ and in this case the even and odd Roy-type NLCS become the usual even and odd coherent states (Hillery, 1987; Xia and Guo, 1989) respectively. However, when $\eta \neq 0$ nonlinearity starts developing, with the degree of nonlinearity depending on the magnitude of the parameter η.

We now turn to the phase probability distributions for the even and odd Roy-type NLCS given by Eq. (6). According to the Pegg–Barnett phase operator formalism (Barnett and Pegg, 1989; Pegg and Barnett, 1988, 1989) we start with a finite dimensional $(s+1)$ Hilbert space spanned by the number states $|0\rangle, |1\rangle, \ldots, |s\rangle$. In this space a complete orthonormal set of phase states $|\theta_m\rangle$, $m = 0, 1, 2, \ldots, s$, is defined by

$$|\theta_m\rangle = \frac{1}{\sqrt{s+1}} \sum_{n=0}^{s} e^{in\theta_m} |n\rangle, \tag{8}$$

where θ_m are given by

$$\theta_m = \theta_0 + \frac{2m\pi}{s+1}, \qquad m = 0, 1, 2, \ldots, s. \tag{9}$$

The value of θ_0 is arbitrary and defines a particular basis in the phase space. In this space a hermitian phase operator Φ_θ is defined as

$$\Phi_\theta = \sum_{m=0}^{s} \theta_m |\theta_m\rangle\langle\theta_m|. \tag{10}$$

For superposition states of the form $|\psi\rangle = \sum_{n=0}^{\infty} b_n e^{in\varphi}|n\rangle$ the phase probability distribution is given by

$$|\langle\theta_m|\psi\rangle|^2 = \frac{1}{s+1} + \frac{2}{s+1}\sum_{n>k} b_n b_k \cos[(n-k)(\varphi - \theta_m)]. \tag{11}$$

Choosing θ_0 as

$$\theta_0 = \varphi - \frac{s\pi}{s+1}, \tag{12}$$

we obtain from Eq. (10)

$$|\langle\theta_m|\psi\rangle|^2 = \frac{1}{s+1} + \frac{2}{s+1}\sum_{n>k} b_n b_k \cos\left[(n-k)\frac{2\mu\pi}{s+1}\right], \tag{13}$$

Where $\mu = m - s/2$. The continuous phase probability distribution $P(\theta)$ can now be obtained as

$$\begin{aligned} P(\theta) &= \lim_{s\to\infty} \frac{s+1}{2\pi}|\langle\theta_m|\psi\rangle|^2 \\ &= \frac{1}{2\pi}\left(1 + 2\sum_{n>k} b_n b_k \cos[(n-k)\theta]\right), \quad (-\pi \le \theta \le \pi). \end{aligned} \tag{14}$$

For the even and odd Roy-type NLCS given by (6), the continuous phase probability distribution $P_\pm(\theta)$ is given by

$$P_\pm(\theta) = \frac{1}{2\pi}\left(1 + 2\sum_{n>k} (b_\pm)_n (b_\pm)_k \cos[(n-k)\theta]\right), \quad (-\pi \le \theta \le \pi), \tag{15}$$

where

$$(b_\pm)_n = N_\pm N_f \frac{[r^n \pm (-r)^n]\, f(n)!}{\sqrt{n!}} \tag{16}$$

The results of numerical computations of the continuous phase probability distribution for the even and odd Roy-type NLCS are presented in Figs. 1–4, respectively.

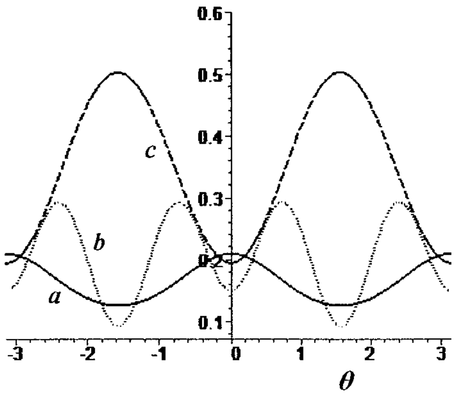

Fig. 1. Phase distribution of the even Roy-type NLCS for $\beta = 0.4$ and $\eta = 0.3$ (curve a), 0.6 (curve b), and 0.8 (curve c).

In Figs. 1 and 2, for the even and odd Roy-type NLCS, we plot the phase probability distribution against θ keeping β fixed ($\beta = 0.4$) and using different values of $\eta(\eta = 0.3, 0.6,$ and $0.8)$ for the three curves. For the even Roy-type NLCS, from Fig. 1 we can see that for small value of η the distribution has only a central peak at $\theta = 0$ (see curve a). As η increased the central peak disappears and four peaks develop at $\theta = \pm\pi/4$, $\theta = \pm 3\pi/4$ (see curve b), or two peaks develop at $\theta = \pm\pi/2$ (see curve c), and these peaks become prominent. Thus the even Roy-type NLCS quantum interference effects become prominent for relatively

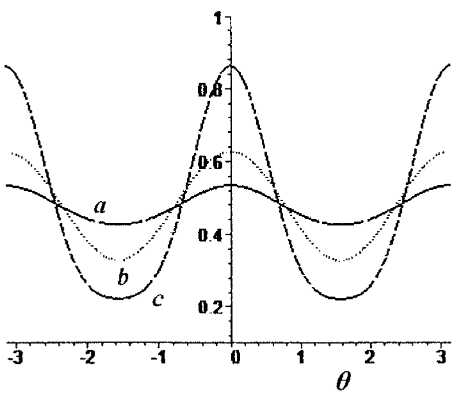

Fig. 2. Phase distribution of the odd Roy-type NLCS for $\beta = 0.4$ and $\eta = 0.3$ (curve a), 0.6 (curve b), and 0.8 (curve c).

large values of η. On the other hand, for the odd Roy-type NLCS, from Fig. 2 we find that for different values of $\eta(\eta = 0.3,\ 0.6,$ and 0.8) the distribution has only a central peak at $\theta = 0$. The qualitative feature of the distribution remains essentially the same for different values of η. However, it may be noted that as η increases the peak structure becomes more and more prominent. In addition, from Figs. 1 and 2 it is clear that the distribution for the even and odd Roy-type NLCS are rather different, and unlike the case of ordinary even and odd coherent states the Pegg–Barnett distribution clearly reflects the different character of quantum interference in the case of the even and odd Roy-type NLCS.

In Figs. 3 and 4, for the even and odd Roy-type NLCS, we plot the phase probability distribution keeping η fixed at 0.3 and varying $\alpha(\alpha = 0.2, 0.4,$ and 0.6). From the figures it is seen that the phase probability distribution of the even and odd Roy-type NLCS is the formally same (see Figs. 3 and 4), and the qualitative feature of the distribution remain formally similar when η is kept fixed while β varies. However, it may be noted that as β increases the peak structure becomes more and more prominent.

4. CONCLUSIONS

It may be noted that the phase probability distribution is an essential tool in the study of various phase characteristics. In this paper, on the basis of our recent work (Wang *et al.*, 2003), using the Pegg–Barnett formalism (Barnett and Pegg, 1989; Pegg and Barnett, 1988, 1989) of phase operator, we investigated the phase probability distributions of the new even and odd NLCS (i.e., the even and odd Roy-type NLCS). It is shown that the distributions for the even and odd

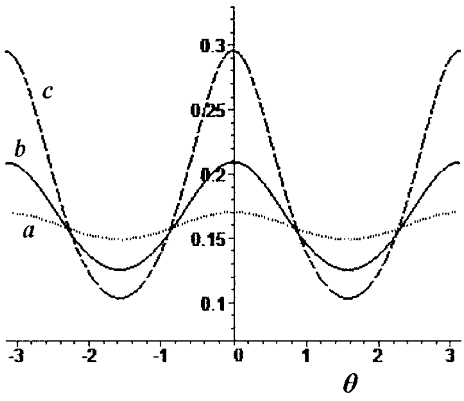

Fig. 3. Phase distribution of the even Roy-type NLCS for $\eta = 0.3$ and $\beta = 0.2$ (curve a), 0.4 (curve b), and 0.6 (curve c).

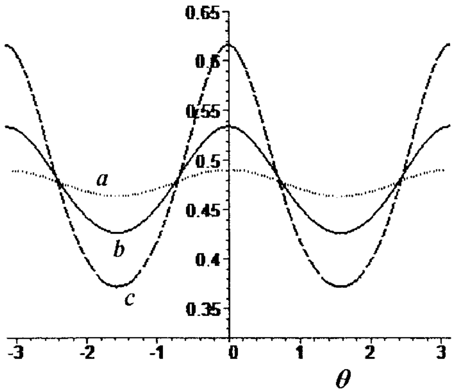

Fig. 4. Phase distribution of the odd Roy-type NLCS for $\eta = 0.3$ and $\beta = 0.2$ (curve *a*), 0.4 (curve *b*), and 0.6 (curve *c*).

Roy-type NLCS are rather different, and unlike the case of ordinary even and odd coherent states the Pegg–Barnett distribution clearly reflects the different character of quantum interference in the case of the even and odd Roy-type NLCS.

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